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Deformation in Three Dimensional Thermoelastic Medium with One Relaxation Time

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The present investigation concerned with deformation in three dimensional thermoelastic medium due to thermal source by finite element method. A particular type of thermal source has been taken to illustrate the utility of the approach. The components of displacement, stress and temperatutre change are obtained and depicted graphically for a specific model.

Keywords: Three Dimensional, Generalized Thermoelasticity, Relaxation Time, Finite Element Method.

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1. INTRODUCTION

The classical theory of uncoupled thermoelasticity predicts two phenomena not compatible with physical observation. First, the heat conduction equation of this theory does not contain any elastic terms contrary to the fact that the elastic changes produce heat effects. Second, the heat conduction equation is of parabolic type, predicting an infinite speed of propagation for heat waves. To overcome the first shortcoming, Biot[2] introduced coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled thermoelasticity of Biot's based on the Fourier's law of heat conduction can predict an infinite speed of heat propagation only.

Whereafter, most of the approaches that came out to overcome the unacceptable prediction of the classical theory are based on the general notion of relaxing the heat flux in the classical Fourier heat conduction equation, thereby introducing a non-Fourier effect. One of the generalized theory of thermoelasticity was presented by Lord and Shulman (L-S)[3] which is also referred as extended thermoelasticity theory, by modifying the Fourier's law of heat conduction with the introduction of a thermal relaxation time parameter. The modified heat conduction equation in this theory is of the wave type and it ensures the finite speeds of propagation of heat and elastic waves. This theory was extended by Dhaliwal and Sherif[4] to general anisotropic media in the presence of heat sources. The uniqueness of solution for this theory was proved under different conditions by Ignaczak,[5,6] by Dhaliwal and Sherief[7] and by Sherief.[8]

Most of the thermoelasticity, magneto-thermoelasticity (generalized or coupled) problems have been solved by using potential functions. This method isnot always suitable as discussed by Sherief[9] and Sherief and Anwar.[10] These may be summarized by (i) the boundary and initial conditions for physical problems are directly related to the physical quantities under consideration and not to the potential function and (ii) the solution of the physical problem in natural variables is convergent while other potential function representations is not convergent always. The alternative to the potential function approach are-(i) State-Space approach: This method is essentially an expansion in a series in terms of the coefficient matrix of the field variables in ascending powers and applying Caley-Hamilton theorem, which requires extensive algebra, and (ii) Eigenvalue approach: This method reduces the problem on vector-matrix differential equation to an algebraic eigenvalue problems and the solutions for the field variables are achieved by determining the eigenvalues and the corresponding eigenvectors of the coefficient matrix. In eigenvalue approach the physical quantities are directly involved in the formulation of the problem and as such the boundary and initial conditions can be applied directly. Body forces and/or heat sources are also accommodated in both the theories, see Das et al.[11] Lahiri et al.[12] Kar et al.[13]

Now-a-days many methods are available for solving linear differential equations. Each method has advantages and disadvantages. The well-known analytical methods like Laplace, Fourier, Hankel transforms, separation of variables, Greens' function etc. are not always useful for numerical calculations, as some problems with numerical convergence of series occurs, and more over shape

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of the body has to be simple. Recently many researchers are using finite element method in different branches in thermoelastic fields like coupled and generalized thermoelasticity,[14–16] bending analysis of micropolar elastic beam by Ref. [17], and modeling of Piezoelectric sensors and actuators by Refs. [18, 19]. Although finite element method are very useful in engineering but not suitable for solving inverse problems. Cialkowaski et al.[20] shows that finite element method gives good solutions of inverse problems only if the internal responses are given in the first row of elements. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods.[21–29] Recently,[30–32] variants problems in waves are studied. Other forms are described for example in the Refs. [33–36].

The current topic is concerned with three dimensional problem for a homogeneous, isotropic and thermoelastic half-space subjected to time dependent heat source on the free surface. Finite element technique has been used to solve the non-dimensional equation. Numerical results for temperature, thermal stress and displacement distributions are presented graphically and discussed.

2. GOVERNING EQUATION

Equation of motion:

$$\sigma_{ij,j} = \rho \ddot{u}_i \tag{1}$$

Heat conduction equation is:

$$k\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho c_E T + \gamma T_0 e) \tag{2}$$

Stress-displacement-temperature relations:

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma(T - T_0)\delta_{ij} \tag{3}$$

where $i, j = 1, 2, 3$ refer to general coordinates.

In the preceding equations, λ and μ are Lame's constant, ρ is the density, σ_{ij} are the component of stress tensor, u_i are the component of displacement vector, t is the time variable, T is the absolute temperature, γ is the material constant given by $\gamma = (3\lambda + 2\mu)\alpha_t$ where α_t is the coefficient of linear thermal expansion, K is a material constant, characteristic of the theory, C_E is the specific heat at constant strain, T_0 is the temperature of the medium in its natural state, assumed to be such that $|(T - T_0)/T_0| \ll 1$.

3. FORMULATION OF THE PROBLEM

The equations of motion are:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} \tag{4}$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \tag{5}$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \tag{6}$$

The Stress-displacement-temperature relations are:

$$\sigma_{xx} = (\lambda + 2\mu)u_{,x} + \lambda(v_{,y} + w_{,z}) - \gamma(T - T_0) \tag{7}$$

$$\sigma_{yy} = (\lambda + 2\mu)v_{,y} + \lambda(u_{,x} + w_{,z}) - \gamma(T - T_0) \tag{8}$$

$$\sigma_{zz} = (\lambda + 2\mu)w_{,z} + \lambda(u_{,x} + v_{,y}) - \gamma(T - T_0) \tag{9}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{10}$$

$$\sigma_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \tag{11}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{12}$$

To transform the above equations in non-dimensional forms, we define the following non-dimensional variables:

$$(x', y', z') = c_0\eta(x, y, z), \quad (u', v', w') = c_0\eta(u, v, w)$$

$$(t', \tau_0') = c_0^2\eta(t, \tau_0), \quad T' = \frac{\gamma(T - T_0)}{\rho c_0^2}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\mu}$$

$$\eta = \frac{\rho c_E}{k}$$

Equations (4)–(12) in the non-dimensional forms then reduce to

$$\beta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + (\beta^2 - 1)\frac{\partial^2 v}{\partial x \partial y} + (\beta^2 - 1)\frac{\partial^2 w}{\partial x \partial z} - \beta^2 \frac{\partial T}{\partial x} = \beta^2 \frac{\partial^2 u}{\partial t^2} \tag{13}$$

$$\beta^2 \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + (\beta^2 - 1)\frac{\partial^2 u}{\partial x \partial y} + (\beta^2 - 1)\frac{\partial^2 w}{\partial y \partial z} - \beta^2 \frac{\partial T}{\partial y} = \beta^2 \frac{\partial^2 v}{\partial t^2} \tag{14}$$

$$\beta^2 \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + (\beta^2 - 1)\frac{\partial^2 u}{\partial x \partial z} + (\beta^2 - 1)\frac{\partial^2 v}{\partial y \partial z} - \beta^2 \frac{\partial T}{\partial z} = \beta^2 \frac{\partial^2 w}{\partial t^2} \tag{15}$$

$$\nabla^2 \theta = (\dot{T} + \tau_0 \ddot{T}) + \varepsilon(\dot{e} + \tau_0 \ddot{e}) \tag{16}$$

$$\sigma_{xx} = 2\frac{\partial u}{\partial x} + (\beta^2 - 2)e - \beta^2 T \tag{17}$$

$$\sigma_{yy} = 2\frac{\partial v}{\partial y} + (\beta^2 - 2)e - \beta^2 T \tag{18}$$

$$\sigma_{zz} = 2\frac{\partial w}{\partial z} + (\beta^2 - 2)e - \beta^2 T \tag{19}$$

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$$\sigma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{20}$$

$$\sigma_{yz} = \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \tag{21}$$

$$\sigma_{xy} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{22}$$

where

$$\beta^2 = \frac{c_0^2}{c_2^2}, \quad c_0^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad \varepsilon = \frac{\gamma^2 T_0}{\rho c_E(\lambda + 2\mu)}$$

and we have dropped the prime for our simplicity.

4. NORMAL MODE ANALYSIS

Solution of the physical variables can be decomposed in terms of normal modes in the following form:

$$[u, v, w, T, \sigma_{ij}](x, y, z, t) = [u^*, v^*, w^*, T^*, \sigma_{ij}^*]e^{\omega t + iay + ibz} \tag{23}$$

where $i = \sqrt{-1}$, ω is the angular frequency and a, b are the wave numbers in the y and z-directions respectively.

Using Eq. (23), we can obtain the following equations from (13)–(22) respectively

$$\frac{d^2u^*}{dx^2} = a_1\frac{dv^*}{dx} + a_2\frac{dw^*}{dx} + a_3\frac{dT^*}{dx} + a_4 u^* \tag{24}$$

$$\frac{d^2v^*}{dx^2} = b_1\frac{\partial u^*}{\partial x} + b_2 v^* + b_3 w^* + b_4 T^* \tag{25}$$

$$\frac{d^2w^*}{dx^2} = c_1\frac{du^*}{dx} + c_3 v^* + c_3 w^* + c_4 T^* \tag{26}$$

$$\frac{d^2T^*}{dx^2} = d_1\frac{\partial u^*}{\partial x} + d_2 v^* + d_3 w^* + d_4 T^* \tag{27}$$

$$\sigma_{xx}^* = 2\frac{\partial u^*}{\partial x} + (\beta^2 - 2)\left[\frac{\partial u^*}{\partial x} + iav^* + ibw^*\right] - \beta^2 T^* \tag{28}$$

$$\sigma_{yy}^* = 2iav^* + (\beta^2 - 2)\left[\frac{\partial u^*}{\partial x} + iav^* + ibw^*\right] - \beta^2 T^* \tag{29}$$

$$\sigma_{zz}^* = 2ibw^* + (\beta^2 - 2)\left[\frac{\partial u^*}{\partial x} + iav^* + ibw^*\right] - \beta^2 T^* \tag{30}$$

$$\sigma_{xy}^* = \frac{dv^*}{dx} + iau^* \tag{31}$$

$$\sigma_{yz}^* = ibv^* + iaw^* \tag{32}$$

$$\sigma_{zx}^* = \frac{dw^*}{dx} + ibu^* \tag{33}$$

where,

$$a_1 = -\frac{ia(\beta^2 - 1)}{\beta^2}, \quad a_2 = -\frac{ib(\beta^2 - 1)}{\beta^2}$$
$$a_3 = 1, \quad a_4 = \frac{(a^2 + b^2 + \beta^2\omega^2)}{\beta^2} \tag{34}$$

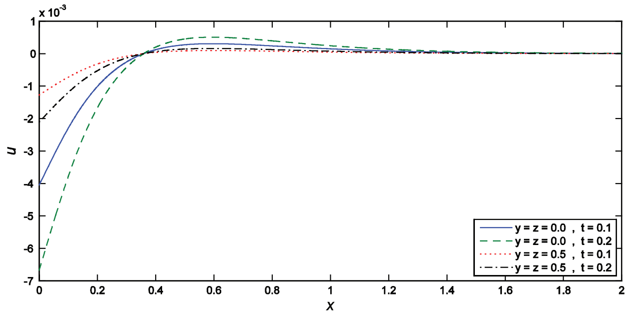

Fig. 1. The displacement distribution for different values of y, z and t.

$$b_1 = -ia(\beta^2 - 1), \quad b_2 = (\beta^2 a^2 + b^2 + \beta^2\omega^2)$$
$$b_3 = [(\beta^2 - 1)ab], \quad b_4 = \beta^2 ia \tag{35}$$

$$c_1 = -ib(\beta^2 - 1), \quad c_2 = [(\beta^2 - 1)ab]$$
$$c_3 = (a^2 + b^2\beta^2 + \beta^2\omega^2), \quad c_4 = \beta^2 ib \tag{36}$$

$$d_1 = \varepsilon\omega^\alpha, \quad d_2 = ia\varepsilon\omega^\alpha, \quad d_3 = ib\varepsilon\omega^\alpha, \quad d_4 = [a^2 + b^2] \tag{37}$$

5. APPLICATION

We consider the following boundary conditions at the surface $x = 0$.

(a) Mechanical boundary condition that the bounding plane to the surface $x = 0$ has no traction anywhere, so we have

$$\sigma(0, y, z, t) = \sigma_{xx}(0, y, z, t) = \sigma_{yy}(0, y, z, t) = \sigma_{zz}(0, y, z, t) \tag{38}$$

which gives

$$\sigma^*(x) = \sigma_{xx}^*(x) = \sigma_{yy}^*(x) = \sigma_{zz}^*(x) \text{ on } x = 0 \tag{39}$$

(b) The thermal boundary condition is

$$q_n + \nu T = r(0, y, z, t) \tag{40}$$

where q_n denotes the normal components of the heat flux vector, ν is the Biot's number and $r(0, y, z, t)$ represents the intensity of the applied heat sources. In order to use the thermal boundary condition (40), we use the generalized

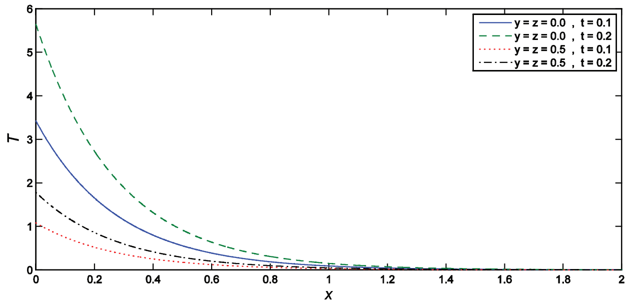

Fig. 2. The temperature distribution for different values of y, z and t.

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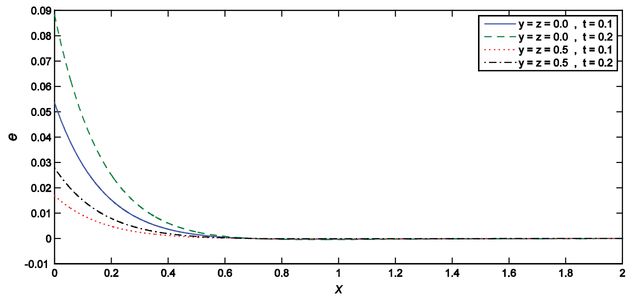

Fig. 3. The strain distribution for different values of y, z and t.

Fourier's law of heat conduction in the non-dimensional form, namely

$$r(1+\tau_0\frac{\partial}{\partial t})q_n = -\frac{\partial T}{\partial n} \tag{41}$$

From Eqs. (39)–(41), we get

$$\nu T - \frac{1}{(1+\tau_0 w)}\frac{\partial T}{\partial x} = r^* \text{ on } x=0 \tag{42}$$

6. NUMERICAL SCHEME

The solution of the ordinary differential Eqs. (24)–(27) is a numerical solution. Therefore, the finite element method has been implemented for solving these with the boundary conditions in the application. The finite element technique is an efficient technique for solving the ordinary or partial differential equations. This method is so general that it can be applied to a wide variety of engineering problems including heat transfer, fluid, mechanics, chemical processing etc. For the finite element method one can refer to Abbas and his colleges.[37–49] The weak formulations of the non-dimensional governing equations are derived. Next, the governing equations are multiplied by independent weighting functions and are then integrated over the spatial domain including the boundary. Applying integration by parts and making use of the divergence theorem reduces the order of the spatial derivatives and allows for the application of the boundary conditions using the well-known Galerkin procedure. The unknown fields u, v, w and T and the corresponding weighting functions $\delta u, \delta v, \delta w$ and δT are approximated by the same shape functions. The last step towards the finite element discretization is to choose the element type and the associated shape functions. Thus, the finite element equations corresponding to (24–27) can be obtained as

$$\int_\Gamma \delta u\left(\frac{d^2u^*}{dx^2} - a_1\frac{dv^*}{dx} - a_2\frac{dw^*}{dx} - a_3\frac{dT^*}{dx} - a_4u^*\right)dx = 0$$

$$\int_\Gamma \delta v\left(\frac{d^2v^*}{dx^2} - b_1\frac{\partial u^*}{\partial x} - b_2v^* - b_3w^* - b_4T^*\right)dx = 0$$

$$\int_\Gamma \delta w\left(\frac{d^2w^*}{dx^2} - c_1\frac{du^*}{dx} - c_3v^* - c_3w^* - c_4T^*\right)dx = 0$$

$$\int_\Gamma \delta T\left(\frac{d^2T^*}{dx^2} - d_1\frac{\partial u^*}{\partial x} - d_2v^* - d_3w^* - d_4T^*\right)dx = 0$$

7. NUMERICAL EXAMPLE AND DISCUSSIONS

For numerical computation, we take the following values of the parameters

$$\lambda = 7.76\times 10^{10}\ (\text{kg})(\text{m})^{-1}\ (\text{s})^{-2}$$
$$\mu = 3.86\times 10^{10}\ (\text{kg})(\text{m})^{-1}\ (\text{s})^{-2};\quad T_0 = 293\ (\text{K})$$
$$K = 3.68\times 10^{2}\ (\text{kg})(\text{m})(\text{K})^{-1}\ (\text{s})^{-3}$$
$$\alpha_t = 17.8\times 10^{-6}\ (\text{K})^{-1};\quad r^* = 100;\quad \nu = 50$$
$$\omega = 5;\quad \tau_0 = 0.2$$

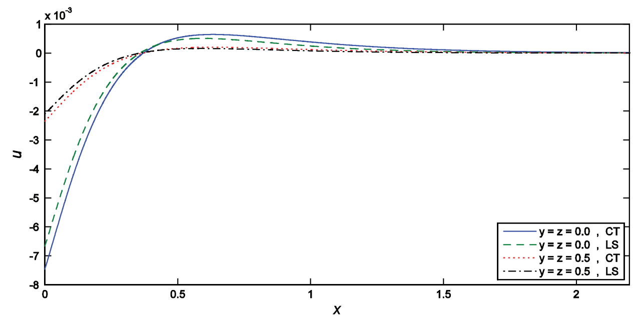

Fig. 5. The displacement distribution for different theories.

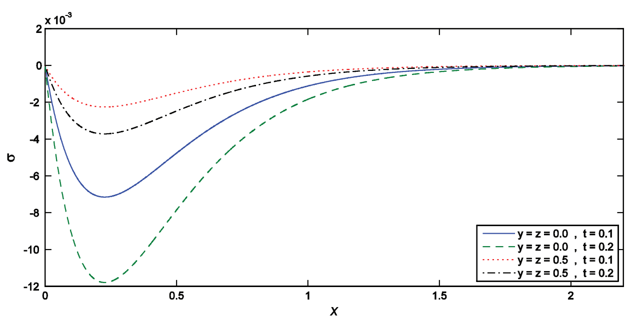

Fig. 4. The stress distribution for different values of y, z and t.

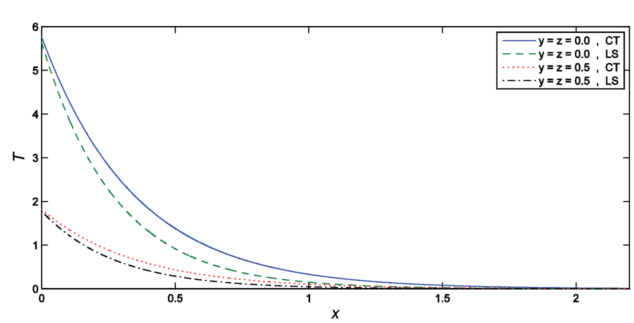

Fig. 6. The temperature distribution for different theories.

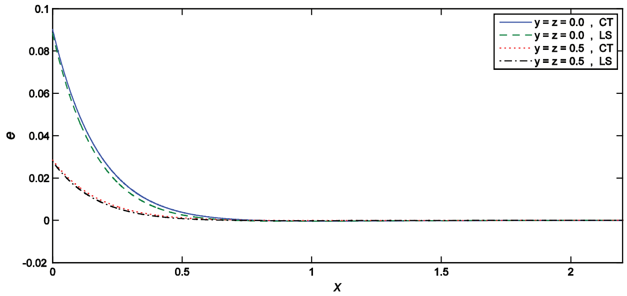

Fig. 7. The strain distribution for different theories.

8. DISCUSSION

Figures 1–4 shows the variation of displacement (u), Temperature (T), strain (e) and stress (σ) with change in x for different cases:

It is clear from Figure 1 that for $0 \leq x < 0.4$, displacement (u) for all the cases attains negative values and increase continuously and outside this region displacement becomes steady and for $x > 1.2$ it is identical for all the cases. Also in the region $0 \leq x < 0.4$, displacement is more for non-zero values of y and z and decreases with increase in the value of relaxation time t.

Figure 2 depicts that temperature distribution (T), decrease continuously till it becomes constant at $x = 1.2$. Its value is large when y and z takes zero value and also takes higher value at $t = 0.2$ as compared to $t = 0.1$. For $x \geq 1.2$, all cases becomes identical and attains minimum value.

It is clear from Figure 3 that variation in the values of strain e is similar to that of T in Figure 2 but magnitude values of e are less in comparison to T. In addition, when $x \geq 0.6$ all the curves coincide and shows no variation. Also, its magnitude values are more for $t = 0.2$ as compared to $t = 0.1$. In addition, the values of T are more for $y = z = 0$.

Figure 4 shows stress (σ), decrease initially for $x \leq 0.2$ but increase continuously for other values until at becomes steady and in absence of the effect of different cases. Its values are more for $t = 0.1$ as compared to $t = 0.2$. Also its value remains more at non-zero values of y and z.

Figure 5 shows that initial increase in values but shows no change for large values of x. Also initially for non-zero values of y and z, displacement (u) remains more but shows opposite behavior for large values of x and becomes steady and same for all the cases when $x \geq 1.5$. Also values of u for LS theory are more as compared to CT theory.

Figure 6 depicts that absolute temperature (T) decrease continuously for all the cases and remains more for CT theory as compared to LS theory. Also the values of T are more for $y = z = 0$. Also for $x \geq 1.5$, it is invariable.

Figure 7 shows that strain change (e) also decrease continuously till it becomes constant. The effect of CT and LS theories is minimum on e. The values of e are also more for $y = z = 0$ as compared to their nonzero values.

Figure 8 shows that stress (σ) initially decrease and then increase till it becomes constant for large values of x. Its values are more for non zero values of y and z and also for LS theory as compared to CT theory.

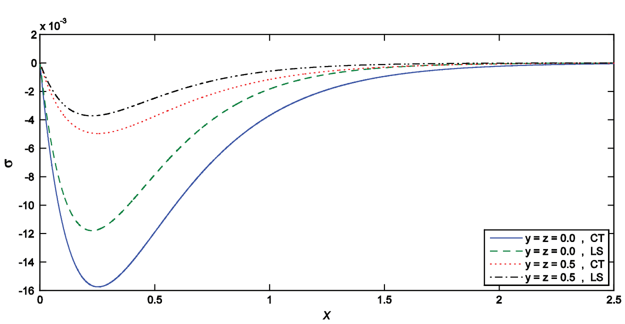

Fig. 8. The stress distribution for different theories.

9. CONCLUSION

Finite element technique the three dimensional problem for a homogeneous, isotropic and thermoelastic half-space is studied subjected to time dependent heat source on the free surface. The effect of relaxation times and thermoelastic theories (CT, LS) and the values of y and z is plotted quantities is significant. The values of displacement (u) and stress (σ) remains more for LS theory and decrease as relaxation time is increased whereas values of absolute temperature (T) and strain (e) are more for CT theory and increase as relaxation time is decreased. Also the values of displacement (u) and stress (σ) remains more for non-zero values of y and z as compared to the values of absolute temperature (T) and strain (e) which remains more for $y = z = 0$.

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